



THE
ABEL
PRIZE
2018

The Norwegian Academy of Science and Letters has decided
to award the Abel Prize for 2018 to

Robert P. Langlands

of the Institute for Advanced Study, Princeton, USA,

“for his visionary program connecting
representation theory to number theory.”

The Langlands program predicts the existence of a tight web of connections between automorphic forms and Galois groups.

The great achievement of algebraic number theory in the first third of the 20th century was class field theory. This theory is a vast generalisation of Gauss’s law of quadratic reciprocity. It provides an array of powerful tools for studying problems governed by abelian Galois groups. The non-abelian case turns out to be substantially deeper. Langlands, in a famous letter to André Weil in 1967, outlined a far-reaching program that revolutionised the understanding of this problem.

Langlands’s recognition that one should relate representations of Galois groups to automorphic forms involves an unexpected and fundamental insight, now called Langlands functoriality. The key tenet of Langlands functoriality is that automorphic representations of a reductive group should be related, via L-functions, to Galois representations in a dual group.

Jacquet and Langlands were able to establish a first case of functoriality for $GL(2)$, using the Selberg trace formula. Langlands’s work on base change for $GL(2)$ proved further cases of functoriality, which played a role in Wiles’s proof of important cases of the Shimura-Taniyama-Weil conjecture.

The group $GL(2)$ is the simplest example of a non-abelian reductive group. To proceed to the general case, Langlands saw the need for a stable trace formula, now established by Arthur. Together with Ngô’s proof of the so-called Fundamental Lemma, conjectured by Langlands, this has led to the endoscopic classification of automorphic representations of classical groups, in terms of those of general linear groups.

Functoriality dramatically unifies a number of important results, including the modularity of elliptic curves and the proof of the Sato-Tate conjecture. It also lends weight to many outstanding conjectures, such as the Ramanujan-Petersson and Selberg conjectures, and the Hasse-Weil conjecture for zeta functions.

Functoriality for reductive groups over number fields remains out of reach, but great progress has been achieved by the work of many experts, including the Fields medallists Drinfeld, Lafforgue and Ngô, all inspired by the guiding light of the Langlands program. New facets of the theory have evolved, such as the Langlands conjectures over local fields and function fields, and the geometric Langlands program. Langlands’s ideas have elevated automorphic representations to a profound role in other areas of mathematics, far beyond the wildest dreams of early pioneers such as Weyl and Harish-Chandra.

