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# Pierre Deligne

Pierre Deligne was born on 3 October 1944 in Etterbeek, Brussels, Belgium. He is Professor Emeritus in the School of Mathematics at the Institute for Advanced Study in Princeton, New Jersey, USA. Deligne came to Princeton in 1984 from Institut des Hautes Études Scientifiques (IHÉS) at Bures-sur-Yvette near Paris, France, where he was appointed its youngest ever permanent member in 1970.

When Deligne was around 12 years of age, he started to read his brother's university math books and to demand explanations. His interest prompted a high-school math teacher, J. Nijs, to lend him several volumes of "Elements of Mathematics" by Nicolas Bourbaki, the pseudonymous grey eminence that called for a renovation of French mathematics. This was not the kind of reading matter that one would normally dream of offering a 14-year old, but for Deligne it became a life changing experience. From then on he never looked back.

Although his father wanted him to become an engineer and to pursue a career that would afford him a good living, Deligne knew early on that he should do what he loved, and what he loved was mathematics. He studied mathematics at the Université Libre de Bruxelles (University of Brussels) and received his Licence en mathématiques, the equivalent of a B.A., in 1966 and his Ph.D., Doctorat en mathématiques, in 1968. In 1972, Deligne received the doctorat d'État ès Sciences Mathématiques from Université Paris-Sud 11.

Deligne went to the University of Brussels with the ambition of becoming a high-school teacher, and of pursuing mathematics

as a hobby for his own personal enjoyment. There, as a student of Jacques Tits, Deligne was pleased to discover that, as he says, "one could earn one's living by playing, i.e. by doing research in mathematics."

After a year at École Normal Supérieure in Paris as auditeur libre, Deligne was concurrently a junior scientist at the Belgian National Fund for Scientific Research and a guest at the Institut des Hautes Études Scientifiques (IHÉS). Deligne was a visiting member at IHÉS from 1968-70, at which time he was appointed a permanent member.

Concurrently, he was a Member (1972-73, 1977) and Visitor (1981) in the School of Mathematics at the Institute for Advanced Study. He was appointed to a faculty position there in 1984.

Pierre Deligne is a research mathematician who has excelled in making connections between various fields of mathematics. His research has led to several important discoveries. One of his most famous contributions was his proof of the Weil conjectures in 1973. This earned him both the Fields Medal (1978) and the Crafoord Prize (1988), the latter jointly with Alexandre Grothendieck. Deligne was awarded the Balzan Prize in 2004 and Wolf Prize in 2008.

When Deligne was awarded the Fields Medal, David Mumford and John Tate, both at the Harvard Mathematics Department, wrote in Science magazine that "There are few mathematical subjects that Deligne's questions and comments do not clarify, for he combines powerful technique, broad knowledge, daring imagination, and unflinching instinct for the key idea."

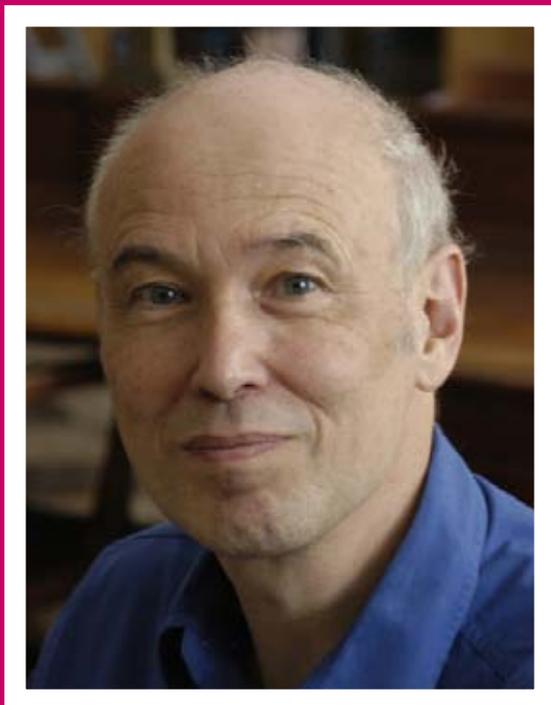


PHOTO: Cliff Moore

Abel Laureate 2013

## Pierre Deligne

Institute for Advanced Study in Princeton, New Jersey, USA

**“for seminal contributions to algebraic geometry  
and for their transformative impact on number theory,  
representation theory, and related fields”**

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# Pierre Deligne and the Weil conjectures

Arne B. Sletsjøe

Deligne's best known achievement is his spectacular solution of the last and deepest of the Weil conjectures, namely the analogue of the Riemann hypothesis for algebraic varieties over a finite field. André Weil wrote in 1949, in the paper *Numbers of solutions of equations in finite fields: ... and other examples which we cannot discuss here, seem to lend some support to the following conjectural statements, which are known to be true for curves, but which I have not so far been able to prove for varieties of higher dimension.*"

The statements Weil was not able to prove have been named the Weil conjectures. The issue of the Weil conjectures is so-called zeta functions. Zeta functions are mathematical constructions that keep track of the number of solutions of an equation, in different number systems. When Weil says that the conjectural statements are known to be true for curves, he means that they are true for equations in two unknown. Varieties in higher dimensions, as referred to, correspond to equations in three or more unknowns.

The equation  $x^2 - y^2 = 3$  describes a plane curve, and in the inset *Counting modulo 5* we have showed that the equation has 4 solutions in the number system  $\{0, 1, 2, 3, 4\}$  when counting modulo 5.

We notice that none of the numbers 0, 1, 2, 3, 4 has square equal to 2. We therefore introduce a new number  $\alpha$ , the square root of 2. This number is not an element of the original set 0, 1, 2, 3, 4 and is determined by the equation  $\alpha^2 = 2$ . Extending the number system to include  $\alpha$ , gives us many

new solutions to the equation  $x^2 - y^2 = 3$ , e.g.  $x = 0$  and  $y = \alpha$  since  $0^2 - \alpha^2 = -2 = 3$  when counting modulo 5. Another solution is given by  $x = \alpha$  and  $y = 2$ . All together we find 24 different solutions in the extended number system. The two numbers 4 and 24 decides the two first terms of the zeta function in the example.

The Weil conjectures are formulated in four statements. Weil proved himself the conjectures in the curve case. For more general equations, three of the four statements were proved by other mathematicians in the following 10-15 years after the publishing of Weil's paper in 1949. The last statement, the most difficult, analogous to the Riemann hypothesis, was proved by Pierre Deligne in 1974.

Soon after the announcement of the conjectures it was clear that they would be proved to be true if one could find a certain type of cohomology, called Weil cohomology. Cohomology are mathematical tools that were developed in 1920- and 30's to understand and systematize knowledge about geometric shapes and structures. The more complicated the structure, the more cohomology. Weil had no suggestions on how to define Weil cohomology, but he knew what qualities cohomology should have to provide a proof of the Weil conjectures.

At the end of the 1940s nobody knew any cohomology which could solve the conjectural problem and thus unify the geometric aspect, related to the solution of equations and the arithmetic aspect, represented by the finite fields (number sys-

tems). The solution came in 1960. At that time Alexander Grothendieck introduced the concept of étale cohomology and proposed that it should play the role of the mysterious, unknown, but essential Weil cohomology. The problem however, was to prove that the étale cohomology satisfy the requirements to be a Weil cohomology. Grothendieck was not able to do so, but

fortunately he had a young student, Pierre Deligne, who succeeded in this task. By a complicated argument, where he based his arguments on several previous achievements made by other mathematicians, Deligne was able to prove the Weil conjectures in full generality. The result provoked attention and brought Deligne into the mathematical elite.

**The first one leads**

In 2006 Pierre Deligne was ennobled by the Belgian king as viscount. In connection with his viscount honour, Deligne designed a coat of arms inspired by the song “As three hens head for the fields”. The song is a tautology, Deligne explained, “and one can view mathematics as being also (long) chains of tautologies.”

**As three hens head for the fields,  
The first one leads,  
The second follows the first,  
The third one is last.  
As three hens head for the fields,  
The first one leads.**



**Counting modulo 5**

Counting modulo 5 means that instead of counting 0,1,2,3,4,5,6,7,8,..., we count 0,1,2,3,4,0,1,2,3,..., i.e. we start again at 0 every time we reach 5. The computation 4+2 means counting 2 steps further from 4. Counting modulo 5, doesn't bring us to 6, but rather to 1, i.e.. 4+2=1. The computation 3·4 modulo 5, means counting to 4 three times, i.e. 1,2,3,4,0,1,2,3,4,0,1,2, which gives 3·4=2. The number system {0,1,2,3,4} with these computation rules is called a **finite field** of 5 elements.

Our aim is to find the solutions of the equation  $x^2-y^2=3$  within this number system. Computing all the squares,  $0^2=0$ ,  $1^2=1$ ,  $2^2=4$ ,  $3^2=4$ , and  $4^2=1$ , we see that the only possibility to achieve a difference 3 between two squares is when  $x^2=4$  and  $y^2=1$ . There are two numbers of square 4

( $2^2$  and  $3^2$ ) and two of square 1 ( $1^2$  and  $4^2$ ), thus we get all together four solutions,  $x=2$  and  $y=1$ ,  $x=2$  and  $y=4$ ,  $x=3$  and  $y=1$ , and  $x=3$  and  $y=4$ .

An example of modulo-counting is time, where we count modulo 12. If we leave home at 10 o'clock and stay out for four hours, then we return at 2 o'clock.

	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>0</b>	0	0	0	0	0
<b>1</b>	0	1	2	3	4
<b>2</b>	0	2	4	1	3
<b>3</b>	0	3	1	4	2
<b>4</b>	0	4	3	2	1

**Multiplication table, modulo 5**

## About the Abel Prize

The Abel Prize is an international award for outstanding scientific work in the field of mathematics, including mathematical aspects of computer science, mathematical physics, probability, numerical analysis, scientific computing, statistics, and also applications of mathematics in the sciences. The Norwegian Academy of Science and Letters awards the Abel Prize based upon recommendations from the Abel Committee.

The Prize is named after the exceptional Norwegian mathematician Niels Henrik Abel, born in 1802. Upon his early death at the age of 26, he left a large body of work, including the proof of the impossibility of solving the general quintic equation by radicals.

The Niels Henrik Abel Memorial Fund was established by the Norwegian Government on January 1, 2002. According to the statutes, the objective is both to award the annual Abel Prize, and to contribute towards raising the status of mathematics in society

and stimulating the interest of children and young people in mathematics. The prize amounts to 6 million NOK (about 800,000 Euro or about 1 million USD) and was first awarded in 2003.

The Norwegian Academy of Science and Letters appoints the Abel Board which is responsible for the activities related to the Abel Prize, and which supports The Abel Symposia, The International Mathematical Union's Commission for Developing Countries, and The Bernt Michael Holmboe Memorial Prize for excellence in teaching mathematics in Norway. In addition, the Abel Board funds national mathematical contests and various projects and activities in order to stimulate interest in mathematics among children and youth. The Norwegian Academy of Science and Letters co-organizes the annual Abel Conference at the Institute for Mathematics and its Applications in Minnesota.



Abel celebration in Oslo.



Peter Lax meets enthusiastic school children in Bergen.

For more information see [www.abelprize.no](http://www.abelprize.no)

# Abel Prize Laureates



2013

**Pierre Deligne**

"for seminal contributions to algebraic geometry and for their transformative impact on number theory, representation theory, and related fields."



2012

**Endre Szemerédi**

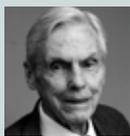
"for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory."



2011

**John Milnor**

"for pioneering discoveries in topology, geometry and algebra."



2010

**John Torrence Tate**

"for his vast and lasting impact on the theory of numbers."



2009

**Mikhail Leonidovich Gromov**

"for his revolutionary contributions to geometry."



2008

**John Griggs Thompson and Jacques Tits**

"for their profound achievements in algebra and in particular for shaping modern group theory."



2007

**Srinivasa S. R. Varadhan**

"for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations."



2006

**Lennart Carleson**

"for his profound and seminal contributions to harmonic analysis and the theory of smooth dynamical systems."



2005

**Peter D. Lax**

"for his groundbreaking contributions to the theory and application of partial differential equations and to the computation of their solutions."



2004

**Sir Michael Francis Atiyah and Isadore M. Singer**

"for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics."



2003

**Jean-Pierre Serre**

"for playing a key role in shaping the modern form of many parts of mathematics, including topology, algebraic geometry and number theory."

