Abel Prize 2008

”For shaping modern group theory”

The scientific committee responsible for choosing the Abel Prize laureates argues for *John G. Thompson* and *Jacques Tits* by referring to their achievements in *shaping modern group theory*. For a non-expert it is diffucult, or in fact impossible, to judge their choice. Group theory is a subject that is usually first taught to third year undergraduate students. In this note the aim is to present group theory in general, with focus on the contributions of Thompson and Tits. Hopefully non-experts can have at least a small glimpse of this beautiful world.

Most people will agree on what it is to have a perfect symmetry. A human face has perfect symmetry if the two halves are mirror images of one another. A square has several symmetries; there are several symmetry axes which generate perfect reflections of the two halves. Denote such symmetries mirror symmetries. But there are other symmetries of the square. Putting a needle in the middle of the square and rotating by an angle of 90 degrees will produce a rotation symmetry. The common property of symmetry is that you do something to the object which preserves its shape.

The possibility of combining symmetries and produce new symmetries is of great importance. Rotating twice by 90 degrees is the same as rotating by 180 degrees. Reflecting a square along a vertical axis, followed by a reflection along a horizontal axis again produce a rotation by 180 degrees. Two reflections give a rotation! This is definitely not the same as reflecting twice along the same axis, since in that case we end up back where we started. Group theory is the formalisation of this game.

Another illustration of group theory is the following. If you go to bed at 11 pm and sleep for 8 hours, what time is it when you wake up? Maybe an easy exercise, but the point is not why it is 7 o’clock in the morning, the question is rather why it is not 18 o’clock. The answer is rather obvious; this is not the way it works. Hours belong to the set {0,1,2,3,..,11} and passing 12 we start over again. The set {0,1,2,…,11} with this rule is a cyclic group. Group theory is the study of structures like these.

Group theory is a fairly old member of the family of mathematical sciences. When ***Niels Henrik Abel*** and ***Evariste Galois*** in the years 1822-32 independently worked on the problem of finding which polynomial equations could be solved by radicals, they laid in principle the foundation of group theory. The concept of an abstract group was not fully understood until 50 years later.

At that time group theory was given another corner stone. Inspired by the work of Abel the two mathematician ***Sophus Lie*** and ***Felix Klein*** approached group theory from a geometric point of view, as we described in the example with the symmetries of the square. Klein tried to describe geometric objects through their symmetries, wheras Lie focused on more general objects like curves and surfaces, and his concept of symmetry was wider, as he allowed certain types of stretching of the objects. Today these transformation groups are known as Lie groups.

John Thompson and Jacques Tits have in different ways continued to work along the lines set up by Abel and Lie; Thompson on finite groups following Abel and Tits on linear groups in the footsteps of Lie.

Betweeen Abel and Lie and Thompson and Tits, a whole century of research on group theory passed by, and one could maybe think that the field might be exhausted for results. This is not the case, and the results of Thompson and Tits are nice evidences thereof. They have both, in their own ways, presented deep and groundbreaking contributions to our understanding of group theory: Through new constructions they have enriched our view of the beauty of group theory and through achievements they have provided us with new insight on the structures on which the theory is based, hence made the field continously interesting for further research. The common base for the results of Thompson and Tits is their originality and deep understanding, which has had an enormous influence on the development of algebra and its related subfields.