

Abel Prize 2008

Why John G. Thompson is awarded the Abel prize for 2008



The citation of the Abel Committee gives the official reason for awarding the Abel Prize to John G. Thompson. Here are some more details for non-experts.

The Abel Committee writes in the citation for the Abel Prize for 2008 about **John Griggs Thompson**:

Thompson revolutionised the theory of finite groups by proving extraordinarily deep theorems that laid the foundation for the complete classification of finite simple groups, one of the greatest achievements of twentieth century mathematics.

What the Committee refers to here is the huge international project known as the Classification of Finite Simple Groups, initiated during the fifties and concluded around 1980. **Daniel Gorenstein**, who took an overview of the whole project and steered it to a successful conclusion, wrote later: *In February 1981 the classification of finite simple groups was completed.*

Few people could imagine that this stage of the project could be reached within three decades. The simple groups are the atoms of group theory; being simple, they form the building blocks for all other groups. A classification of all groups therefore has to start with the simple groups.

At the International Congress of Mathematicians in Amsterdam in 1954, **Richard Brauer** suggested a strategy for the completion of this project. The concluding remark of Gorenstein referred to one of the most remarkable proofs in the history of mathematics. The proof covers more than 10 000 pages, 500 papers by more than 100 mathematicians from all over the world. Here is a short version of the theorem:

Classification Theorem for finite simple groups

Any finite simple group belongs to one of the following three families: cyclic groups

of prime order, alternating groups or finite groups of Lie type, or the group is one of the 26 sporadic groups

The major breakthrough in the classification project came in 1962. Quoting the Committee:

In a major breakthrough, Feit and Thompson proved that every non-elementary simple group has an even number of elements.

Equivalently,

every finite group of odd order is soluble.

This is a nice example of the fact that, very often, the simplest is also the best (and most difficult): The theorem can be formulated using 8 words, it can be understood by an undergraduate student in mathematics, the original proof filled an entire volume of the *The Pacific Journal of Mathematics* (255 pages), and the result caused a revolution in modern group theory. Thompson continued working along these lines. His next theorem gave a classification of so-called N-groups. The Committee writes:

Later Thompson extended this result to establish a classification of an important kind of finite simple group called an N-group. At this point, the classification project came within reach

The visionary speaker at the Amsterdam Congress reached a similar conclusion when he once more gave a speech about the project, this time at the International Congress in Nice in 1970. Richard Brauer then said:

Up to the early 1960s, really nothing of real interest was known about general simple groups of finite order.... Since [1962], finite group theory simply is not the same any more.

Thompson continued his work for the classification project and his interest was focused on the 26 sporadic groups. The first 5 of these groups were found at an early stage in group theory, namely by *Émile Mathieu* in the 1860s. Mathieu's groups were not particularly big. The largest one, M_{24} , has order 244 823 040, a small number when it comes to sporadic groups. The largest sporadic group was named *The Monster* by *John Conway* and has order

808017424794512875886459904
961710757005754368000000000

One of the other sporadic groups has got its name after John Thompson. It is called the *Thompson group*, abbreviated *Th*. The order of *Th* is 90745943887872000, also relatively small

compared to the Monster. The Committee refers to Thompson's work on the sporadic groups in its citation:

Its almost incredible conclusion is that all finite simple groups belong to certain standard families, except for 26 sporadic groups. Thompson and his students played a major role in understanding the fascinating properties of these sporadic groups, including the largest, the so-called Monster.

Thompson is by many referred to as the world's leading group theorist. His name is forever tied to the huge classification project, or the *Thirty Years War* as Daniel Gorenstein called it. But the highlight is the Feit-Thompson theorem, so compactly formulated that it is worthwhile repeating:

Every finite group of odd order is soluble.